Reg No.:

Name :

U8345

University of Kerala

First Semester Degree Examination, November 2024

Four Year Undergraduate Programme

Discipline Specific Course

Mathematics

UK1DSCMAT100, Foundations of Mathematics

Academic Level: 100-199

Time: 2 hours **Max. Marks: 56**

Part A. 6 Marks. Time:5 Minutes
Objective Type. 1 Mark Each. Answer all Questions
(Cognitive Level: Remember/Understand)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
1.	Define an anti-symmetric relation.	Remember	CO4
2.	A matrix A is said to be non singular if..	Remember	CO1
3.	The determinant of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is	Understand	CO1
4.	A homogeneous linear system of n equations with n unknowns has a unique solution if...	Understand	CO2
5.	The linear congruence $ax \equiv b(\mod m)$ has a unique solution if and only if	Remember	CO3
6.	The sum, $\sum_{i=1}^{n}(2i-1)$ is	Remember	CO3

Part B. 10 Marks. Time:20 Minutes
Two-Three sentences. 2 Marks Each. Answer all Questions
(Cognitive Level: Remember/Understand/Apply)

Qn. No.	**Question**	**Cognitive Level**	**Course Outcome (CO)**
7.	Define one-to-one function. Give an example.	Remember	CO4
8.	Show that for any square matrix A, $\frac{1}{2}(A+A^t)$ is always symmetric, where A^t is the transpose of A.	Remember	CO1
9.	Express $(28, 12)$ as a linear combination of 28 and 12.	Remember	CO3
10.	Find gcd of 120 and 28.	Understand	CO3
11.	State Rouche's theorem. Give an example of a system of equations which is inconsistent.	Apply	CO2

Part C. 16 Marks. Time:35 Minutes
Short-Answer. 4 Marks Each. Answer all Questions, choosing among options within each question.
(Cognitive Level: Understand/Analyse/Apply)

Qn. No.	**Question**	**Cognitive Level**	**Course Outcome (CO)**
12.	A.)Show that the relation $\equiv$ is an equivalence relation in the set of all integers. **OR** B.) Define congruence relation. The equivalence relation $\equiv$ on the set of integers defined by xRy if $x \equiv y(\mod 4)$. Find all equivalence classes under this relation.	Understand	CO4
13.	A.) If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, in which a, b, c are different, show that $abc = 1$. **OR** B.) Express $\begin{bmatrix} 3 & 5 & -7 \\ -8 & 11 & 4 \\ 13 & -14 & 6 \end{bmatrix}$ as the sum of a lower triangular matrix with zero leading diagonal and an upper triangular matrix.	Apply	CO1

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
14.	A.) For what value of λ, the system of equation $$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = 1$$ has unique solution? **OR** B.) Find the values of k for which the system of equations $$\begin{aligned}(3k-8)x + 3y + 3z &= 0\\ 3x + (3k-8)y + 3z &= 0\\ 3x + 3y + (3k-8)z &= 0.\end{aligned}$$ has a non-trivial solution.	Analyse	CO2
15.	A.)Find the remainder when 3^{181} is divided by 17. **OR** B.) Using canonical decomposition of 1050 and 2574, find their lcm.	Understand	CO3

Part D. 24 Marks. Time:60 Minutes

Long-Answer. 6 Marks Each. Answer all 4 Questions, choosing among options within each question. (Cognitive Level: Understand/Analyse/ Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	A) Find the number of positive integers in the range 1976 through 3776 that are; (i.) Divisible by 13 or 15. (ii.) Not divisible by 15 or 17. **OR** B) Using Euclidean algorithm find $(4076, 1024)$ and express $(4076, 1024)$ as a linear combination of 4076 and 1024.	Understand	CO3

17.	A.) Find the values of a and b for which the equations $$x + ay + z = 3, x + 2y + 2z = b, x + 5y + 3z = 9$$ are consistent. When these equations have a unique solution? **OR** B.) Test the consistency and if possible solve $$\begin{aligned} 4x + 2y + z + 3w &= 0 \\ 6x + 3y + 4z + 7w &= 0 \\ 2x + y + w &= 0. \end{aligned}$$	Understand	CO2
18.	A.) Determine the values of p such that the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ p & 2 & 2 & 2 \\ 9 & 9 & p & 3 \end{bmatrix}$ is 3. **OR** B.) Using Gauss-Jordan method find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.	Analyse	CO1
19.	A.) Let the functions f and g defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find $f \circ f, f \circ g$, and $g \circ f$. **OR** B.)Define partial ordering. What is the difference between an equivalence relation and a partial ordering. Show that the relation $\leq$ on the set of all real numbers is a partial ordering.	Apply	CO4